Section 2.5

Math 231

Hope College

1. Given an $n \times n$ matrix A, a matrix B is called the **inverse** of A if
$$AB = BA = I_n.$$
2. The inverse does not exist for every matrix A. For example, the matrix $\mathbf{0}$ has no inverse.
3. When the inverse of a matrix A does exist, it is unique, and is denoted A^{-1}. (See Theorem 2.42.)
4. **Theorem 2.44:** If A and B are invertible matrices of the same size, then AB is invertible and
$$(AB)^{-1} = B^{-1}A^{-1}.$$
5. **Theorem 2.48:** If $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is an $n \times n$ linear system with A invertible, then the full (unique) solution is $\vec{\mathbf{x}} = A^{-1}\vec{\mathbf{b}}$.

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Inverses of 2×2 Matrices

1. There is a simple formula for finding inverses of 2×2 matrices.

   Theorem 2.46: Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2×2 matrix. If $ad - bc \neq 0$, then A is invertible and

   $$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

2. Finding inverses of larger matrices can be accomplished through Gauss-Jordan Elimination. Set up the augmented matrix $(A|I_n)$ and row-reduce. If the result to the left of the bar is I_n, then the final matrix will be $(I_n|A^{-1})$.

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